Pearson Edexcel

Model Solutions

Additional Assessment Materials
Summer 2021

Pearson Edexcel GCE in Mathematics
9MA0 (Public release version)

Resource Set 1: Topic 1
Proof

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General guidance to Additional Assessment Materials for use in 2021

Context

- Additional Assessment Materials are being produced for GCSE, AS and A levels (with the exception of Art and Design).
- The Additional Assessment Materials presented in this booklet are an optional part of the range of evidence teachers may use when deciding on a candidate's grade.
- 2021 Additional Assessment Materials have been drawn from previous examination materials, namely past papers.
- Additional Assessment Materials have come from past papers both published (those materials available publicly) and unpublished (those currently under padlock to our centres) presented in a different format to allow teachers to adapt them for use with candidate.

Purpose

- The purpose of this resource to provide qualification-specific sets/groups of questions covering the knowledge, skills and understanding relevant to this Pearson qualification.
- This document should be used in conjunction with the mapping guidance which will map content and/or skills covered within each set of questions.
- These materials are only intended to support the summer 2021 series.

1. Complete the table below. The first one has been done for you.

For each statement below you must state if it is always true, sometimes true or never true, giving a reason in each case.

Statement	Always True	Sometimes True	Never True	Reason
The quadratic equation $ax^2 + bx + c = 0$ ($a \neq 0$) has 2 real roots.		✓		It only has 2 real roots when $b^2 - 4ac > 0$ When $b^2 - 4ac = 0$ it has 1 real root and when $b^2 - 4ac < 0$ it has 0 real roots.
(i) When a real value of x is substituted into $x^2 - 6x + 10$ the result is positive. **(2)**	✓			Completing the square: $(x-3)^2 - 9 + 10$ $(x-3)^2 + 1$ so the minimum y value is 1
(ii) If $ax > b$ then $x > \frac{b}{a}$ **(2)**		✓		True if a is positive but if a is negative $x < \frac{b}{a}$
(iii) The difference between consecutive square numbers is odd. **(2)**	✓			Odd number squared is always odd and an even number squared is always even so the difference is always even

(Total for Question 1 is 6 marks)

2. "If m and n are irrational numbers, where $m \neq n$, then mn is also irrational."
Disprove this statement by means of a counter example.

(2)
(Total for Question 2 is 2 marks)

Take $m = \sqrt{2}$ $n = \sqrt{8}$

So $mn = \sqrt{2}\sqrt{8} = \sqrt{16} = 4$ which is rational

3. Elsa claims that

'for $n \in \mathbb{Z}^+$, if n^2 is even, then n must be even'

Use proof by contradiction to show that Elsa's claim is true.

(2)
(Total for Question 3 is 2 marks)

Assume that if n^2 is even, n can be odd. So $n = 2k + 1$ for $k \in \mathbb{N}$

so $n^2 = (2k+1)^2 = 4k^2 + 4k + 1$
$= 2(2k^2 + 2k) + 1$ which is clearly odd so there is a contradiction. This means if n^2 is even, then n must be even

4. Bobby claims that

$$e^{3x} \geq e^{2x}, \quad x \in \mathbb{R}.$$

Determine whether Bobby's claim is always true, sometimes true or never true, justifying your answer.

(2)

(Total for Question 4 is 2 marks)

Sometimes true. Take $x = 1$, it is clear that $e^3 \geqslant e^2$. Now take $x = -1$ so $\frac{1}{e^3} \geqslant \frac{1}{e^2}$ which is not true as this would imply $e^3 \leq e^2$ so it is sometimes true

5. (i) Prove that for all $n \in \mathbb{N}$, $n^2 + 2$ is not divisible by 4

(4)

(Total for Question 5 is 4 marks)

Even case

Assume n is an even number. This means $n = 2m$ for some $m \in \mathbb{N}$. We now have $(2m)^2 + 2$ $= 4m^2 + 2$. Dividing by 4 we get $m^2 + \frac{1}{2}$ which can never be an integer as m^2 is an integer so the statement is false for n being even.

Odd case

Now assume n is odd so n can be written as $2r + 1$ for some $r \in \mathbb{N}$

Our equation becomes

$$(2r+1)^2 + 2 = 4r^2 + 4r + 1 + 2$$

$$= 4r^2 + 4r + 3$$

when dividing this by 4 we get

$$r^2 + r + \frac{3}{4}$$

$$= r(r+1) + \frac{3}{4}$$

As both r and $r+1$ are integers it is impossible for $r(r+1) + \frac{3}{4}$ to be an integer

$\blacksquare$

6. **In this question you must show all stages of your working.**

A geometric series has common ratio r and first term a.

Given $r \neq 1$ and $a \neq 0$

prove that

$$S_n = \frac{a\left(1-r^n\right)}{1-r}$$

(4)

(Total for Question 6 is 4 marks)

$$S_n = a + ar + ar^2 + ar^3 + \dots + ar^{n-1}$$

$$rS_n = ar + ar^2 + ar^3 + \dots + ar^{n-1} + ar^n$$

$$S_n - rS_n = a - ar^n$$

$$S_n(1-r) = a(1-r^n)$$

$$\Rightarrow \quad S_n = \frac{a(1-r^n)}{(1-r)}$$

$\square$

7. Prove by contradiction that there are no positive integers a and b, with a odd, such that

$$a + 2b = \sqrt{8ab}\,.$$

(Total for Question 7 is 4 marks)

Assume that there are positive integers with
n such that $a+2b=\sqrt{8ab}$

$$a+2b=\sqrt{8ab}$$

$$\Rightarrow (a+2b)^2 = 8ab$$

as a is odd and 2b is even then a+2b is odd so $(a+2b)^2$ is odd. This is a contradiction as 8ab is clearly even.

8. Prove by contradiction that there are no positive integers p and q such that

$$4p^2 - q^2 = 2^5$$

25

(4)

(Total for Question 8 is 4 marks)

Assume there are positive integers such that

$$4p^2 - q^2 = 25$$

$$(2p+q)(2p-q) = 25$$

25 has two factor pairs $(1, 25)$ or $(5, 5)$

So

$$\begin{cases} 2p+q = 5 \\ 2p-q = 5 \end{cases} \Rightarrow 2q = 0 \Rightarrow q = 0$$ which is a contradiction as $q > 0$

or

$$\begin{cases} 2p+q = 25 \\ 2p-q = 1 \end{cases} \Rightarrow 4p = 26 \Rightarrow p = \frac{13}{2}$$ which is a contradiction as p is an integer

9. Use algebra to prove that the square of any natural number is **either** a multiple of 3 **or** one more than a multiple of 3

(4)

(Total for Question 9 is 4 marks)

All natural numbers can be written as

(1) $3n$ (2) $3n+1$ (3) $3n+2$. We prove that the square of all the cases is either divisible by a multiple of 3 or 1 more than a multiple of 3.

(1) $3n$ $(3n)^2 = 9n^2 = 3(3n^2)$ so this case is divisible by 3

(2) $3n+1$ $(3n+1)^2 = 9n^2 + 6n + 1$

$= 3(3n^2 + 2n) + 1$

so it is divisible by 1 more than a multiple of 3

③ $3n+2$

$$(3n+2)^2 = 9n^2 + 12n + 4$$

$$= 3(3n^2+4n) + 3 + 1$$

so again it is 1 more than a multiple of 3